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A STUDY OF ENERGY CONVERSION AND MERIDIONAL CIRCULATION FOR THE LARGE-SCALE MOTION IN THE ATMOSPHERE [1]

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ABSTRACT

Energy conversion between potential and kinetic energy is considered. Section 2 contains the derivations which are necessary to compute the energy conversion for a large region on the basis of vertical velocities and relative topography as obtained from a two-parameter model presently used by the Joint Numerical Weather Prediction Unit. The energy conversion is divided into three parts: (1) energy conversion due to a mean vertical velocity over the region, (2) energy conversion in meridional planes, and (3) energy conversion in the zonal planes.

Section 3 contains a discussion of the results obtained for the months January and April 1959. The energy conversion is positive for each day in both months, but the conversion in the meridional planes has a different sign in the two months, being positive in January and negative in April. The pattern of the mean meridional circulation is discussed and the frictional dissipation estimated.

Section 4 describes a procedure for an evaluation of the energy conversion for the different wave numbers, and discusses the results for the same two months. The same section contains a comparison with results obtained from a linear, adiabatic theory.

Section 5 contains a discussion of the modifications to the results in section 4 caused by the diabatic heating of the atmosphere It is made plausible that the maximum conversion found for the small wave numbers by an adiabatic computation is greatly altered due to the effects of the heating.

1. INTRODUCTION

The problem of conversion between potential and kinetic energy has been studied by numerous authors from Margules' [9] original attempt to account for the kinetic energy of storms to very recent investigations by Phillips [13] and Smagorinsky [16] of the general circulation and the energetics of the atmosphere. These latter investigations are concerned with numerical integrations of idealized models of the atmosphere, including, however, the effects of diabatic heating and friction. A study of energy conversions based on data has been restricted in the past mainly because these conversions depend on the field of vertical motion in the atmosphere. The operational use of simplified baroclinic models of the atmosphere for short-range numerical prediction has made available daily fields of the vertical motion at at least one pressure level in the atmosphere. The vertical motion is of course computed from the model used for the numerical prediction and does not include the effects of heating and friction. It is computed under the assumption that the flow in the atmosphere is quasi-geostrophic, frictionless, and adiabatic. Consequently, it should be treated with great caution.

The vertical velocities computed from the method mentioned above seem nevertheless to be of approximately the correct order of magnitude and with the correct sign as is indicated by the use of the quantity in quantitative precipitation forecasts, for instance. It seems therefore

[1] During the preparation of this study the author became aware of a similar study conducted by Dr. Saltzman and Dr. Fleischer at Massachusetts Institute of Technology Their results for the month of February 1959 agree quite well with those reported here. (Personal communication.)

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worthwhile to use the fields of vertical velocities to obtain estimates of energy conversions. An investigation of this kind was made by White and Saltzman [23] using the vertical velocities from the thermotropic model which was integrated by Gates et al. [5] for the entire month of January 1953. However, the region covered only approximately North America. A later investigation by Palmén [10] used a somewhat larger region, but this computation was made for only a few days in January 1956. The Joint Numerical Weather Prediction (JNWP) Unit has fields of vertical velocities available twice daily since November 1958. The octagonal region used in the numerical prediction covers the Northern Hemisphere down to approximately 13° N. It is these vertical velocities which have been used in the investigation reported in the following sections.

The first problem considered in this paper is the total energy conversion within the octagonal region. In connection with energy conversions it is always interesting to divide the total conversion into at least two parts, the first being due to mean meridional circulations, the other to circulations in the zonal planes. The computations were arranged to give this information together with the mean meridional circulation.

A second problem has been to study the energy conversion as it appears on the different scales of the atmospheric motions.

2. CONVERSION OF POTENTIAL TO KINETIC ENERGY

The computations which have been made assume that the atmosphere is in hydrostatic equilibrium. We are further going to consider only the kinetic energy of the horizontal flow, because the vertical velocity is small compared to the horizontal wind. An equation for the change of kinetic energy may then be obtained from the horizontal equations of motion in the following way. Let us first write the equations of motion with pressure as the vertical coordinate:

$$\frac{\partial u}{\partial t}+\mathbf{V}\cdot\nabla u+\omega\frac{\partial u}{\partial p}=-\frac{\partial\phi}{\partial x}+fv \tag{2.1}$$

$$\frac{\partial v}{\partial t}+\mathbf{V}\cdot\nabla v+\omega\frac{\partial v}{\partial p}=-\frac{\partial\phi}{\partial y}-fu. \tag{2.2}$$

In these equations $\mathbf{V}=\mathbf{V}\ (u,v)$ is the horizontal velocity; $\omega=dp/dt$, the "vertical velocity"; $\phi=gz$, the geopotential; g is the acceleration of gravity; z is the height of an isobaric surface; and f is the Coriolis parameter.

Multiplying (2.1) by u and (2.2) by v, adding the two equations, and defining $k=\frac{1}{2}\ (u^2+v^2)$, we obtain

$$\frac{\partial k}{\partial t}+\mathbf{V}\cdot\nabla k+\omega\frac{\partial k}{\partial p}=-\mathbf{V}\cdot\nabla\phi. \tag{2.3}$$

Integrating (2.3) over the complete volume of the atmosphere and defining the total kinetic energy by:

$$K=\int_0^\infty\int_S \tfrac{1}{2}\rho(u^2+v^2)dSdz=\frac{1}{g}\int_0^{p_0}\int_S kdSdp, \tag{2.4}$$

where ρ is the density and S the area over which we integrate, we obtain

$$\frac{\partial K}{\partial t}+\frac{1}{g}\int_0^{p_0}\oint_L kv_n dldp+\int_0^{p_0}\oint_L zv_n dldp=\int_0^{p_0}\int_S \omega\frac{\partial z}{\partial p}dSdp \tag{2.5}$$

In deriving (2.5) we have used the boundary conditions $\omega=0$ for $p=0$ and $p=p_0$. We have further transformed certain area integrals into line integrals using Gauss' theorem $\left(\oint_L\right.$ means a line integral around the closed boundary L of the region S, while v_n is the outward directed normal velocity$\left.\right)$.

The terms on the left side measure the change in kinetic energy of the volume, and the transport of kinetic energy and potential energy across the boundary. The term on the right hand side measures the conversion of potential to kinetic energy within the volume (White and Saltzman [23]). We shall, with Phillips' notation, write

$$\{P,K\}=\int_0^{p_0}\int_S \omega\frac{\partial z}{\partial p}dSdp \tag{2.6}$$

where $\{P, K\}$ means the conversion from potential to kinetic energy.

The expression (2.6) was evaluated in the following way. The vertical velocities available apply at the 600-mb. level according to the model approximations, which further include the assumption that ω has a parabolic distribution with zero points at the 1000- and 200-mb. levels. The derivative was evaluated by finite differences as

$$\frac{\partial z}{\partial p}=-\frac{1}{35}h \tag{2.7}$$

where h is the thickness field between 850 and 500 mb. This approximation for $\partial z/\partial p$ was used only because h was available for each day.

When the two approximations are introduced in (2.6) we arrive at the following formula

$$\{P,K\}=-\frac{32}{21}\int_S \omega hdS \tag{2.8}$$

where ω now is the vertical velocity at 600 mb.

Let us next separate ω and h into their mean values and deviations from these mean values. We define

$$\left.\begin{aligned}h&=\bar{h}+h'\\ \omega&=\bar{\omega}+\omega'\end{aligned}\right\} \tag{2.9}$$

where the mean values are defined as area means:

$$\overline{(\quad)}=\frac{1}{S}\int_S(\quad)\,dS. \tag{2.10}$$

Inserting the expression (2.9) in (2.8) we obtain

$$\{P, K\}=-\frac{32}{21}\bar{\omega}\bar{h}S-\frac{32}{21}\int_S \omega'h'dS. \tag{2.11}$$

The first term in (2.11) represents an energy conversion due to a mean vertical velocity over the region. Now, if S were the complete surface of the sphere we would have $\bar{\omega}=0$, because

$$\bar{\omega}=-\frac{1}{S}\int_0^p \int_S \nabla\cdot\mathbf{V}\, dSdp\equiv 0, \tag{2.12}$$

i. e., the divergence integrated over the sphere is zero. For a restricted region S, as used in these computations, there is no assurance that $\bar{\omega}=0$. We find in general from the data that $\bar{\omega}$ has small positive or negative values. The first term in (2.11) was computed separately.

It is furthermore of interest to divide the last term in (2.11) into energy conversion due to the presence of a mean meridional circulation. We define therefore the mean meridional circulation by the following operator:

$$\widetilde{(\quad)}=\frac{1}{L}\int_0^L (\quad)\, dx \tag{2.13}$$

where L is the length of the latitude circle and x the eastward directed coordinate.

Writing now:

$$\left.\begin{aligned} h'&=\tilde{h}'+h'' \\ \omega'&=\tilde{\omega}'+\omega'' \end{aligned}\right\} \tag{2.14}$$

we obtain

$$\begin{aligned}\{P,K\}&=-\frac{32}{21}\bar{\omega}\bar{h}S-\frac{32}{21}\int_S \tilde{\omega}'\tilde{h}'dS-\frac{32}{21}\int_S \omega''h''dS \\ &=\{P,K\}_1+\{P,K\}_2+\{P,K\}_3. \end{aligned}\tag{2.15}$$

The term $\{P, K\}_2$, the energy conversion in the mean meridional circulation, was also computed separately.

The values of the vertical velocity and the relative topography used in these computations were given at the grid points of a quadratic grid on a polar-stereographic projection (standard latitude 60° N.). Special care has therefore to be taken when we perform the numerical integrations of the different terms in 2.15). Let us consider, as an example, the last integral $\{P, K\}_3$. We have

$$\{P, K\}_3=-\frac{32}{21}\int_S \omega''h''dS\simeq-\frac{32}{21}\sum_i\sum_j \omega''_{ij}h''_{ij}(\Delta S)_e \tag{2.16}$$

where $(\Delta S)_e$ is the area on the earth corresponding to an elementary grid square on the map. Performing the computations on the map leads then to the expression

$$\{P, K\}_3=-\frac{32}{21}\sum_i\sum_j \omega''_{ij}h''_{ij}(1/m^2)(\Delta S)_m \tag{2.17}$$

where m is the map scale factor, $(1+\sin 60°)/(1+\sin \phi)$, and $(\Delta S)_m$ is the square of the grid interval at the standard latitude, in our case $(3.81\times10^5)^2$m.^{2} Similar caution was taken in forming the mean values $\overline{(\quad)}$ and $\widetilde{(\quad)}$ and in evaluating the other integrals.

The three terms $\{P, K\}_i$, $i=1, 2, 3$ in (2.15) may be thought of as conversions due to (1) a mean vertical lifting or sinking of the air mass within the region S, (2) circulations in the meridional plane, and (3) circulations in the zonal plane. The two last terms measure also according to Lorenz [7] transformations between zonal available potential energy and zonal kinetic energy and transformation between eddy available potential energy and eddy kinetic energy, respectively.

3. RESULTS OF THE TOTAL ENERGY CONVERSIONS

The procedure outlined in the preceding section was used to compute the three terms in (2.15) for each day in the months January and April 1959. Only the data applying at 0000 GMT were used in the computations.

Let us first summarize the results obtained for January 1959. Table 1 contains in the first row the total energy conversion in the average for the month in the unit kj. sec.$^{-1}$. In the second row the same results are rewritten in the unit kj. sec.$^{-1}$m.$^{-2}$ by dividing by the total area of the octagonal region (1.97×10^{14} m.^{2}). For comparison with other estimates the numbers in the second row are converted to the c.g.s. units, erg, gm.$^{-1}$, sec. $^{-1}$ and written in the last row.

Now, in considering the numbers presented in table 1 we notice a number of interesting things. The first column shows (see (2.15)) that the initial vertical velocities indicate a mean downward motion. The mean vertical motion was in fact downward on 70 percent of the days going into the computation. We shall not include $\{P, K\}_1$ in our measure of the total energy conversion since this term would be zero if we considered the whole earth. It would, incidentally, be an advantage to impose the constraint $\bar{\omega}=0$ in the numerical forecasts with two-parameter baroclinic models over an almost hemispheric region in order to avoid fictitious changes in total circulation, as shown by the author (Wiin-Nielsen [21]). If $\bar{\omega}\neq0$, as in these computations, we get a significant energy conversion solely due to the mean lifting of sinking, amounting to about 10 percent of the sum of $\{P, K\}_2$ and $\{P, K\}_3$.

The next important quantity to consider is the energy conversion due to the mean meridional circulation. Our region is so large that it includes the whole polar region, the mid-latitudes, and at least part of the Tropics. The

TABLE 1.—*Total energy conversion computed for January 1959.*

Unit	$\{P, K\}_1$	$\{P, K\}_2$	$\{P, K\}_3$
kj. sec.$^{-1}$	-3.14×10^{10}	$+2.04\times10^{10}$	$+28.82\times10^{10}$
kj. sec.$^{-1}$m.$^{-2}$	-1.59×10^{-4}	$+1.03\times10^{-4}$	$+14.60\times10^{-4}$
erg gm.$^{-1}$ sec.$^{-1}$	-0.159	$+0.103$	$+1.46$

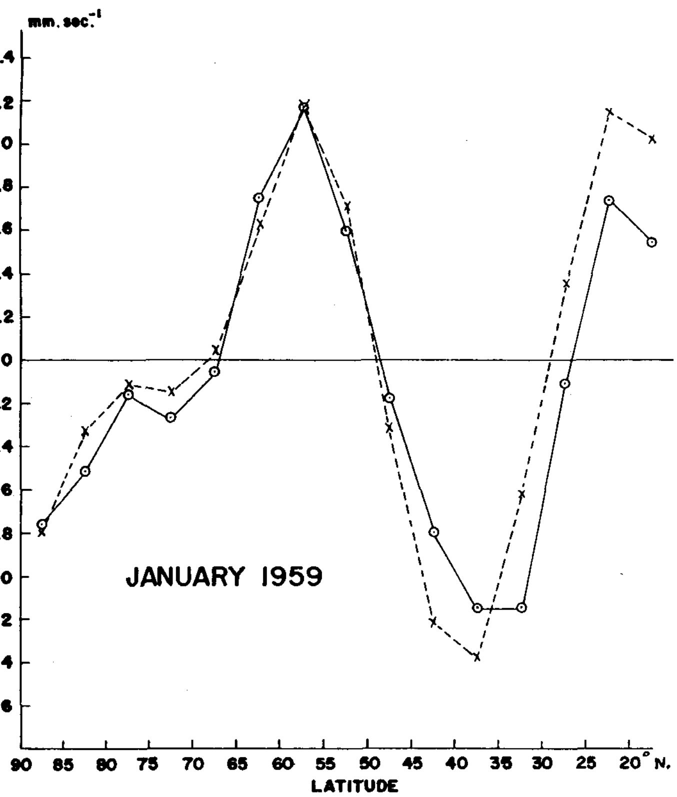

FIGURE 1.—Vertical velocity in units mm. sec.$^{-1}$ averaged along latitudes and in time, for January 1959. The solid curve was computed from observed data, the dashed curve from 36-hour forecasts.

adiabatic method of computing the vertical velocity makes use in our case of the geostrophic wind. This method would certainly break down at and near the equator. As the mean latitude of the southern boundary of the octagon is about 13° N., we can probably believe in the vertical velocities even in the southern portions of the grid. The computation for January shows that we have a positive conversion of potential to kinetic energy in the meridional planes, although it amounts to only about 7 percent of the energy conversion in the zonal planes. It is very interesting first to see what kind of meridional circulation we have in the average for the month. This mean meridional circulation was found by averaging the vertical velocity for each day in the month and then computing the average for the month. The result is given in figure 1, where the horizontal coordinate is latitude from the pole down to 20° N., while the vertical coordinate is the mean vertical velocity in the unit mm. sec.$^{-1}$. Figure 1 shows that the mean meridional circulation for this month definitely consists of the classical three cells with rising motion to the south, the maximum downward motion around 35° N., the maximum upward motion between 55° and 60° N., and downward motion over the polar region.

The dashed curve on figure 1 indicates the mean meridional motion for January 1959 for the 36-hour forecasts. The three-cell pattern is still very well defined in the 36-hour forecasts, but there seems to be a tendency to produce great upward motion in the southernmost portions of the region, and perhaps to shift the maximum downward motion somewhat to the north by about 5° of latitude. Now, with a well-defined indirect cell in the middle latitudes, this cell alone would give a conversion of kinetic energy to potential energy as was also found by White and Saltzman [23] in their computation which uses data for January 1953 between 35° N. and 60° N. The net positive value obtained in this computation shows that the conversion from potential to kinetic energy in the two direct cells to the north and the south is sufficient to give a positive conversion for the whole meridional circulation. It should also be stressed that the main part (93 percent) of the conversion takes place in the zonal planes.

The presence of the three-cell meridional circulations in the initial vertical velocities is especially interesting because these vertical motions are computed from a model which does not include the effect of friction and diabatic heating. It has been pointed out by Phillips [12] in a theoretical study that the three-cell meridional circulation is a result of the presence of baroclinic unstable waves. Phillips also computed the intensity of the vertical velocity in the meridional cross section. In order to make this computation he has estimated an average warming of the northern latitudes of about 0.5° C. day^{-1} in order to balance the yearly average cooling due to radiation over the northern half of the Northern Hemisphere. In this way he arrives at a maximum mean vertical motion of about 3 mb. day^{-1} which, converted to our units, corresponds to about 0.5 mm. sec.$^{-1}$ The computation of the mean meridional circulation made here is certainly in agreement with Phillips' computation as far as the pattern and order of magnitude are concerned, although it seems that the mean meridional circulation is somewhat stronger in the particular month.

The total energy conversion for January 1959 amounts to about 31×10^{10} kj. sec.$^{-1}$ when we add the contribution from the circulations in the meridional and zonal planes. As our region is very large we are probably allowed in the first approximation to neglect the contributions from the advection of kinetic and potential energy into the region; see equation (2.5). If this is the case, we have that the change in total kinetic energy is equal to the conversion from potential to kinetic energy minus the frictional dissipation. Including now the frictional dissipation in equation (2.5) we obtain

$$\frac{\partial K}{\partial t}=\{P, K\}-D \tag{3.1}$$

where D measures the frictional dissipation. In a long-term mean we have $\partial K/\partial t=0$ and the conversion from potential to kinetic energy must balance the frictional dissipation; i.e.,

$$D=\{P,K\} \tag{3.2}$$

If we assume that $\partial K/\partial t=0$ for the month of January 1959, we find that $D=31\times 10^{10}$ kj. sec.$^{-1}$ or about 1.6×10^{-3} kj. m.$^{-2}$ sec.$^{-1}$, a value which agrees rather well with the value estimated by Pisharoty [11], who obtained 2×10^{-3} kj. m.$^{-2}$ sec.$^{-1}$ The value obtained from the present study is about one-third of the estimate made by Brunt [1]: 5×10^{-3} kj. m.$^{-2}$ sec.$^{-1}$ White and Saltzman [23] obtained for the middle latitudes a value closer to Brunt's, while Palmén's [10] recent computation agrees well with the value obtained in this study.

The integrals in (2.5) may be thought of as sums of contributions from different sub-regions of S, although the contribution from a subregion is not related directly to the change of energy within the region because of the contribution from the boundary integrals; i.e., transport of potential and kinetic energy into the region. In the computation of the total energy conversions for the region S, the contribution from each 5-degree latitude ring was computed. We could look upon these values as changes of the kinetic energy, if there were no transport across the boundaries, or simply as measures of the correlation between the vertical motion and the relative topography (i.e., the mean temperature). The contributions from the 5-degree latitude bands are plotted in figure 2, where the horizontal coordinate is latitude and the vertical coordinate has the dimension of an energy conversion per unit area and unit time. As shown by the figure the greatest positive correlation is in the band between 40° and 45° N. and in the polar regions. In these regions therefore in the average warm air is rising and cold air sinking, while this to a much lesser extent is true in the latitude bands 55°–65° N. and 20°–35° N. The northernmost region of these two represents for this month a region where we have a small negative correlation, meaning that in the average the warm air is sinking and the cold air rising. The curve agrees fairly well with the one given by White and Saltzman [23] for the latitude band 35°–60° N.

The computations reported so far give energy conversions as computed from initial data. There are therefore no forecasts involved except for the very first time step which in the present scheme is necessary in order to solve for the vertical velocity in the adiabatic equation. This procedure could actually be avoided by solving the so-called ω-equation (see later). From the point of view of numerical prediction it is of interest to investigate the conversion of potential to kinetic energy as the forecast progresses in time. This investigation may be made along the same lines as outlined in section 2 replacing ω and h by the corresponding forecast values.

The models so far used in short-range prediction have been adiabatic and frictionless. For such a model the energy equation states that the sum of kinetic and potential energy is constant. As the models also have been hydrostatic, the internal energy per unit column is proportional to the potential energy, and the sum of the two

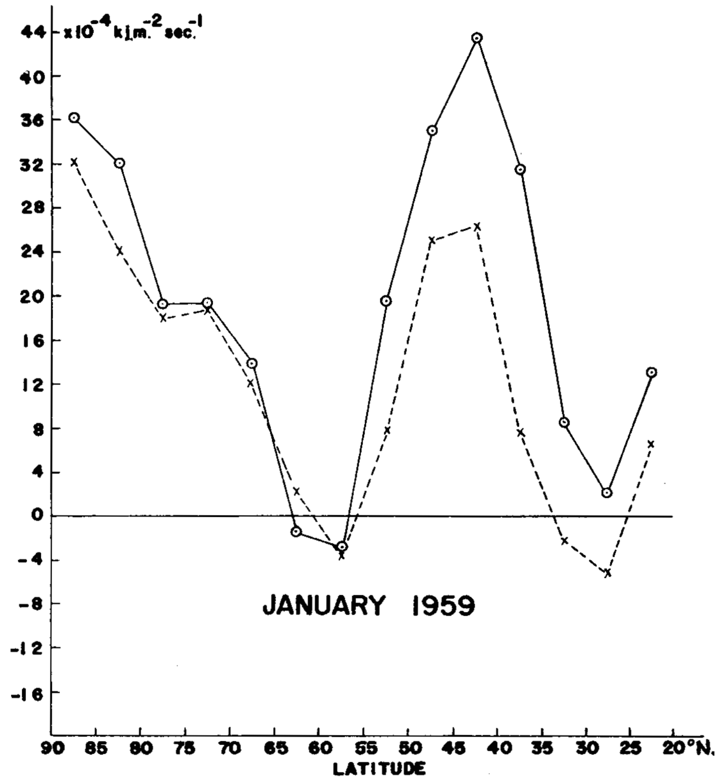

FIGURE 2.—Correlation between vertical velocity and mean temperature expressed as an energy conversion per unit area and unit time as a function of latitude for January 1959. The solid curve was computed from observed data, the dashed curve from 36-hour forecasts.

will be called, as is now customary, the potential energy. As shown by Lorenz [8] the sum of kinetic energy and the available potential energy A is also constant for an adiabatic model, and he estimates that the ratio between kinetic energy and available potential energy is of the order of magnitude of 10^{-1}. One important question is whether the rate of increase of kinetic energy in a model used for numerical prediction is about the same as in the real atmosphere. We shall be able to answer this question by computing the energy conversion at, for instance, 12, 24, and 36 hours under the assumption that the region is so large that it can be considered closed. For simple sinusoidal waves it can be shown that the energy conversion in a quasi-geostrophic model depends upon the phase-lag between the temperature (thickness) field and the pressure field. If the temperature field lags behind the pressure field, as it usually does in the atmosphere, a positive conversion will take place, while the opposite is true in the reverse situation. Now, it has been noted by Thompson [18] that one of the errors in the quasi-geostrophic models is that the phase difference between the temperature field and the pressure field decreases too rapidly as compared to the real atmosphere. We may therefore expect that the energy conversion takes place too rapidly in the models, if the observation above is quite general.

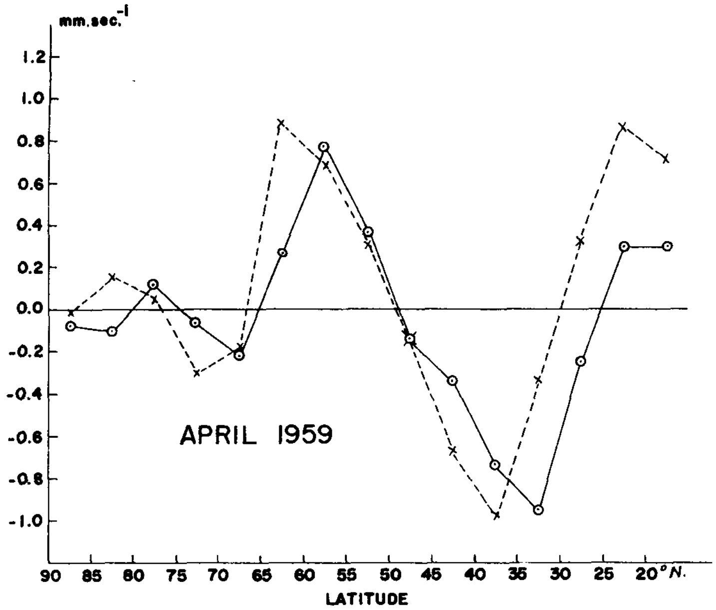

FIGURE 3.—Vertical velocity, April 1959. For explanation see figure 1.

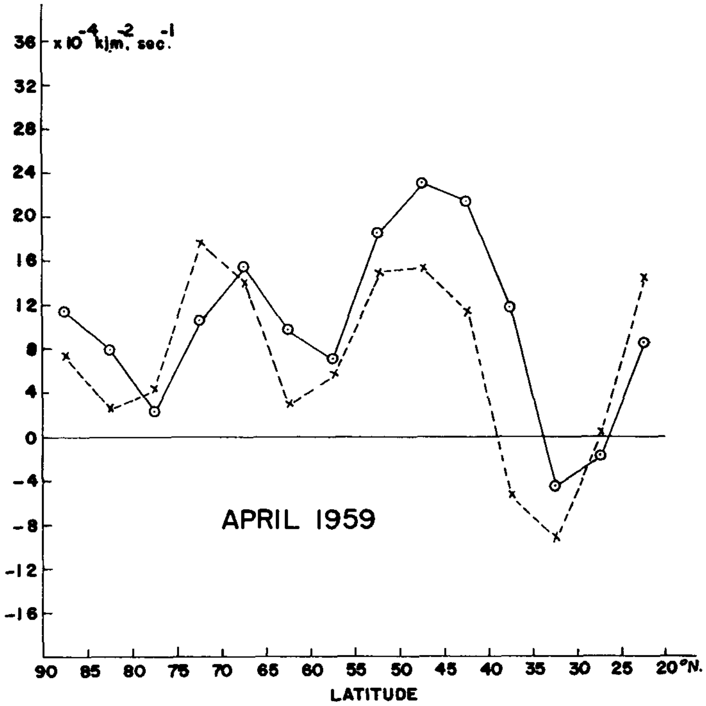

FIGURE 4.—Correlation between vertical velocity and mean temperature, April 1959. For explanation see figure 2.

It is interesting to notice that this is indeed the case. The 36-hour forecast for the month of January has an energy conversion which in total amounts to 24×10^{10} kj. sec.$^{-1}$ as compared to 31×10^{10} kj. sec.$^{-1}$ for the initial data, or roughly ¾ of what it should be. An inspection of the data for the individual days indicates further that there were no days when the conversion computed from the 36-hour forecast was greater than the one in the corresponding initial data. It may therefore be concluded that the conversion of potential energy to kinetic energy in the quasi-geostrophic model gradually decreases as the forecast progresses in time. This fact is also shown clearly in figure 2, where the dashed curve gives the correlation for the month of January between vertical velocity and relative topography in the 36-hour forecasts. Although the general characteristics of the curve are the same, the average level is lower.

In order to investigate whether the same general characteristics would hold for another month the computations were carried out for April 1959. They are summarized in table 2 and in figures 3 and 4.

TABLE 2.—*Total energy conversion computed for April 1959.*

Unit	$\{P, K\}_1$	$\{P, K\}_2$	$\{P, K\}_3$
kj. sec.$^{-1}$	-7.72×10^{10}	-2.12×10^{10}	$+21.66\times10^{10}$
kj. sec.$^{-1}$m.$^{-2}$	-3.92×10^{-4}	-1.08×10^{-4}	$+11.00\times10^{-4}$
erg gm.$^{-1}$sec.$^{-1}$	-0.392	-0.108	$+1.100$

Table 2 shows that we again have a negative contribution from the term $\{P, K\}_1$ indicating a net downward motion over the entire region in the majority of the cases. In April it turns out that an average downward motion exists on all days except one.

Contrary to the situation in January we find now that the mean meridional circulation converts kinetic energy into potential energy. The mean meridional circulation still has the three-cell pattern, although some irregularities now exist in the very high latitudes. The mean vertical motion is weaker in April than in January, but the extreme values appear approximately at the same latitudes in the two months. The main difference between the two months is probably that the direct cells in the low and high latitudes are less developed in April, while the indirect cell in the middle latitudes, although somewhat weaker, still is well defined and regular. These differences account for the different signs of the energy conversion in the meridional plane. However, the energy conversion $\{P, K\}_2$ in April is still a rather small fraction of the conversion in the zonal planes (about 10 percent).

If we, for April, assume that the change of the total kinetic energy is zero ($\partial K/\partial t=0$), we obtain the result that the frictional dissipation for this month amounts to about 1.0×10^{-3}kj. m.$^{-2}$ sec.$^{-1}$ as compared to 1.6×10^{-3} kj. m.$^{-2}$ sec.$^{-1}$ in January.

The correlation between vertical velocity and relative topography, again expressed as an energy conversion per unit area and time, is given in figure 4. We find again that the greatest positive correlation is around 45° N., and that high correlations exist in the high latitudes although more irregularly than in January.

The 36-hour forecast vertical velocity gives the mean meridional circulation drawn as the dashed curve in figure 3. The main error in the 36-hour forecasts for this month is a shift to the north amounting to about 5° of latitude for the maxima and minima, and again we note that the forecast vertical velocities in the low latitudes are somewhat larger than observed. This difference is probably due to erroneous boundary conditions.

The conversion indicated by the 36-hour forecasts is again somewhat smaller in April than the one given by the initial data. While the initial data give a conversion of about 20×10^{10} kj. sec.$^{-1}$ within the octagonal region, we obtain only about 17×10^{10} kj. sec.$^{-1}$ for the 36-hour forecasts, or 85 percent of the value given by the initial data.

4. ENERGY CONVERSIONS ON DIFFERENT SCALES

It has been known for a long time that the atmosphere is not equally effective in releasing potential energy on different scales. According to baroclinic instability theory we find certain scales (4000–6000 km.) which are the most unstable. It is also known that the unstable baroclinic wave converts potential to kinetic energy. The results mentioned above have been obtained from theoretical studies using a linear perturbation theory. An evaluation of the energy conversion on the different scales is therefore worth while doing, first of all to get a check on the validity of the linear perturbation theories, and secondly to investigate whether other energy conversions not treated by the theory could be important. Saltzman [14] has recently considered the energy equation in the wave-number regime. We shall here be interested in only the direct conversion of potential to kinetic energy.

Let us return to the equation (2.8) giving the conversion from potential to kinetic energy:

$$\{P, K\}=-\frac{32}{21}\int_S \omega h dS. \tag{4.1}$$

We shall in the following consider only a sub-region of the complete octagonal region. For convenience we divide the polar-stereographic map into rings having the center at the north pole. The width of the rings is one grid increment on the map. With this arrangement we have:

$$\int_S \omega h dS=\int_0^y \int_0^{2\pi} \omega h R \cos \phi d\lambda dy \tag{4.2}$$

where R is the radius of the earth, considered as a sphere, ϕ is latitude, λ longitude, and y the measure of length in the south-north direction. We may also write (4.2) in the form:

$$\int_S \omega h dS=R\int_0^y I(y) \cos \phi dy \tag{4.3}$$

with

$$I(y)=\int_0^{2\pi} \omega(\lambda, y)h(\lambda, y)d\lambda \tag{4.4}$$

Let us next write $\omega(y)$ and $h(y)$ as Fourier-series in the form:

$$\left.\begin{aligned}\omega(\lambda,y)&=a_0(y)+\sum_{n=1}^{N}\{a_n(y)\sin(n\lambda)+b_n(y)\cos(n\lambda)\}\\ h(\lambda,y)&=A_0(y)+\sum_{n=1}^{N}\{A_n(y)\sin(n\lambda)+B_n(y)\cos(n\lambda)\}\end{aligned}\right\} \tag{4.5}$$

Inserting (4.5) into (4.4) and using the orthogonality of the functions $\cos(n\lambda)$ and $\sin(n\lambda)$ we obtain:

$$I(y)=2\pi a_0(y)A_0(y)+\pi\sum_{n=1}^{N}\{a_n(y)A_n(y)+b_n(y)B_n(y)\}. \tag{4.6}$$

The formulae for the Fourier coefficients are:

$$\left.\begin{aligned}a_0(y)&=\frac{1}{2\pi}\int_0^{2\pi}\omega(\lambda,y)d\lambda\\ a_n(y)&=\frac{1}{\pi}\int_0^{2\pi}\omega(\lambda,y)\sin(n\lambda)d\lambda,\\ b_n(y)&=\frac{1}{\pi}\int_0^{2\pi}\omega(\lambda,y)\cos(n\lambda)d\lambda\end{aligned}\right\} \tag{4.7}$$

and corresponding expressions for $A_0(y)$, $A_n(y)$, $B_n(y)$.

Let us next consider the evaluation of the complete integral. We have

$$\int_0^y I(y)\cos\phi\, dy=\int_0^y I(y)\cos\phi\frac{dS}{m(\phi)} \tag{4.8}$$

where dS now is the distance on the map. Evaluating the last integral by finite differences we obtain

$$\int_0^y I(y)\cos\phi\, dy=\frac{\Delta S}{(1+\sin\phi_0)}\sum_{j=1}^{j_{max}} I_j\cos\phi(1+\sin\phi) \tag{4.9}$$

where we have introduced the expression $m=(1+\sin\phi_0)/(1+\sin\phi)$ and also the symbol j for the counter of the rings. With the present JNWP grid $j_{max}=27$; the counter j is considered to increase from the North Pole toward the equator on the map.

It is now convenient to rewrite the original expression in the form:

$$\{P,K\}=\{P,K\}_{(0)}+\sum_{n=1}^{N}\{P,K\}_{(n)} \tag{4.10}$$

where

$$\left.\begin{aligned}\{P,K\}_{(0)}&=-\frac{32}{21}\frac{\Delta S}{(1+\sin\phi_0)}\times\\ &\qquad\sum_{j=1}^{j_{max}}\{\cos\phi\,(1+\sin\phi)a_0(j)A_0(j)\}\\ \{P,K\}_{(n)}&=-\frac{32}{21}\frac{\Delta S}{(1+\sin\phi_0)}\times\\ &\sum_{j=1}^{j_{max}}\{\cos\phi\,(1+\sin\phi)\,(a_n(j)A_n(j)+b_n(j)B_n(j))\}\end{aligned}\right\} \tag{4.11}$$

The Fourier coefficients were computed using a procedure developed by G. 'Arnason, formerly at JNWP. The arrangement of the grid points in the octagonal grid is not directly suited for a Fourier analysis along latitude circles. A short description of the procedure is therefore necessary.

Each of the zonal bands with a width ΔS is divided into zones of 10° of longitude ($\Delta\lambda=10°$). Exceptions to this are made for the high latitudes. In each zone k we define

$$\left.\begin{aligned}\overline{(\alpha \sin n\,\lambda)_k}&=\frac{1}{N_k}\sum_{i=1}^{N_k}(\alpha\sin n\lambda)_i\\ \overline{(\alpha \cos n\,\lambda)_k}&=\frac{1}{N_k}\sum_{i=1}^{N_k}(\alpha\cos n\lambda)_i\end{aligned}\right\}\qquad(4.12)$$

N_k is the number of grid points in the zone, k. The Fourier coefficients are then computed using the formulae

$$a_n=\frac{1}{18}\sum_{k=1}^{36}\overline{(\alpha\sin n\lambda)_k},\quad b_n=\frac{1}{18}\sum_{k=1}^{36}\overline{(\alpha\cos n\lambda)_k}\qquad(4.13)$$

The increments $\Delta\lambda$ are as follows:

$$\begin{aligned}j\geq 9,&\ \Delta\lambda=10°\\ 9>j\geq 5,&\ \Delta\lambda=20°\\ 5>j\geq 3,&\ \Delta\lambda=30°\\ 3>j>0,&\ \Delta\lambda=60°\end{aligned}$$

where j, as noted earlier, is the counter of grid distances from the pole.

The computations were again made for each day of January and April 1959. The energy conversion was averaged for the month to get a picture of the mean conditions. The results are shown in figures 5 and 6.

For each day the Fourier analysis was made up to wave number 15 to be on the safe side. In the middle latitudes ($\phi=45°$ N.) $n=15$ would correspond to a wavelength of 1800 km., which probably is about the smallest wavelength we can hope to analyze with any accuracy with our present aerological network. It turns out, as can be seen on figures 5 and 6, that the energy conversion is negligible for $n\geq 11$.

The spectra show for both months two rather broad maxima for $n=2$ and $n=6$ in January and $n=2$ and $n=7$ in April. The maximum corresponding to $n=6$ or 7 is clearly connected with the most unstable baroclinic wave. Converting again to wavelength in the middle latitudes we find that $n=6$ and 7 correspond roughly to wavelengths of 4700 and 4000 km., which coincides well with the most unstable waves as predicted from the linear perturbation theory.

It is furthermore predicted by linear perturbation theory that all waves with a wavelength shorter than about 3000 km. for the middle latitudes should be stable. This coincides rather well with the abrupt cutoff in the spectra for $n=10$, which corresponds to $L=2800$ km.

These results support strongly the results of the linear theory. It is in fact not too difficult to see that an adiabatic linear theory must give a result of this nature.

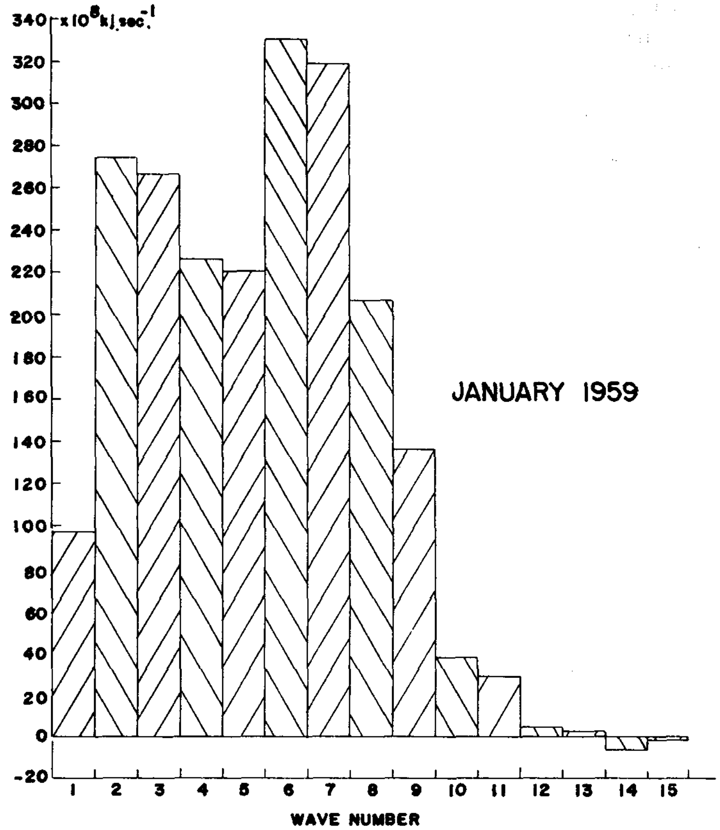

FIGURE 5.—Energy conversion as a function of wave number averaged in time, for January 1959. The horizontal coordinate is number of waves around the hemisphere, while the vertical coordinate is energy conversion per unit time north of about 15° N.

Let us consider a simple sinusoidal wave pattern in the atmosphere. The stream function at 600 mb. and the thermal stream function for the layer 800 to 600, or 600 to 400 mb. will be given in the form

$$\left.\begin{aligned}\psi_2&=-U_2y+A\sin mx\\ \psi'&=-U'y+B\sin(mx+\alpha)\end{aligned}\right\}\qquad(4.14)$$

where U_2 and U' are the zonal winds considered as constants, A and B the amplitudes, $m=2\pi/L$ the wave number, and α the phase-difference between the thermal wave and the stream function. Note that α is positive if the thermal wave is behind the pressure wave.

We want in the following to find the energy conversion for the wave pattern. In order to do this we need the vertical velocity field. This can be found from the ω-equation which for the simple baroclinic model underlying the computations here takes the form

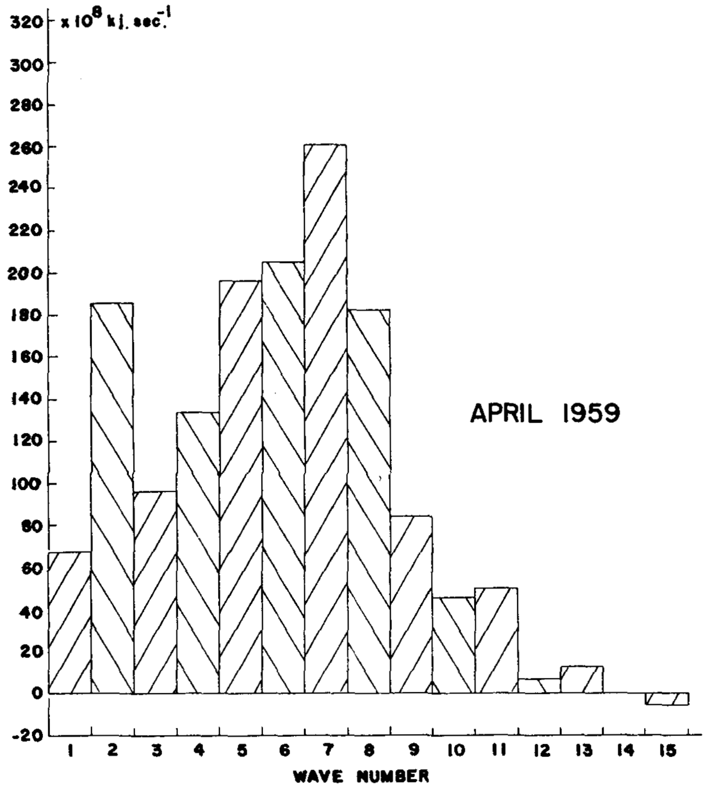

FIGURE 6.—Energy conversion, April 1959. For explanation see figure 5.

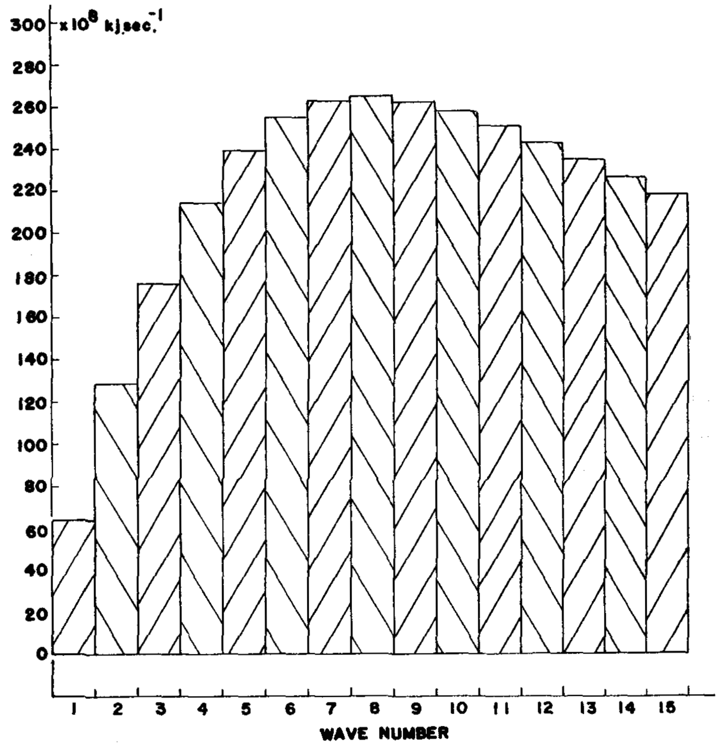

FIGURE 7.—Energy conversion as a function of wave number according to linear adiabatic theory keeping all parameters except the wave number constant.

$$\sigma\nabla^2\omega_2-\frac{2f_0^2}{P^2}\,\omega_2=\frac{2f_0}{P}\{\nabla^2(\mathbf{V}_2\cdot\nabla\psi')-\mathbf{V}_2\cdot\nabla\zeta'-\mathbf{V}'\cdot\nabla(\zeta_2+f)\}\tag{4.15}$$

For the details in this derivation the reader is referred to a paper by the author [21], in which it also is shown that a solution to (4.15) may be written in the form,

$$\omega_2=\frac{2f_0}{P}\,\frac{1}{m^2\sigma+\frac{2f_0^2}{P^2}}\,(\beta v'-2m^2U'v_2)\tag{4.16}$$

for the simple waves given by (4.14). In (4.15) and (4.16) $\sigma=-\alpha\partial\ln\theta/\partial p$ is a measure of static stability, $P=40$ cb., and $v_2=\partial\psi_2/\partial x$, $v'=\partial\psi'/\partial x$ are the meridional velocities in the waves given by (4.14).

The energy conversion per unit area may now easily be computed by inserting the expression for ω_2 and ψ' in the formula

$$\frac{1}{S}\{P,\,K\}=-\frac{32}{21}\,\frac{1}{S}\int_S\omega_2 h\,dS\simeq-\frac{32}{21}\,\frac{1}{S}\,\frac{f_0}{g}\int_S\omega\psi'dS.\tag{4.17}$$

As there is no variation in the y-direction we may take the area S as a unit band; i.e., $S=1\cdot L=2\pi R\cos\phi$. We obtain then

$$\frac{1}{S}\{P,\,K\}=\frac{32}{21}\,\frac{f_0^2}{gP}\,U'v_{2\,max}\,v'_{max}\sin\alpha\;\frac{2m}{m^2\sigma+\frac{2f_0^2}{P^2}}\cdot\tag{4.18}$$

Expressed in this form we find that the energy conversion depends upon (1) the zonal thermal wind, (2) the amplitude of the meridional wind components, (3) the phase-lag between the thermal field and the stream function, and (4) the scale (wave number).

If we next introduce the number of waves n around the hemisphere into (4.18); i.e., $m=n/R\cos\phi$, we obtain:

$$\frac{1}{S}\{P,\,K\}=\frac{32}{21}\,\frac{f_0^2}{gP}\,U'v_{2\,max}\,v'_{max}\sin\alpha R\cos\phi\;\frac{2n}{n^2\sigma+\frac{2f_0^2}{P^2}R^2\cos^2\phi}.\tag{4.19}$$

Several investigations, the first one by Charney [2], have shown that the meridional components (v_2, v') will be greatest for the baroclinic waves ($n=6$ or 7) and for the ultra-long waves ($n=2$ or 3). The wind components above in (4.19) will therefore tend to give a maximum for the wave numbers mentioned above. Beside this there is a definite influence of the scale of the motion as expressed by the last factor in (4.19). If we for a moment assume that $v_{2\,max}$ and v'_{max} are the same for all waves

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we can investigate the influence of the scale alone. Inserting typical values ($U'=5$ m. sec.$^{-1}$, $v_{2\,max}=v'_{max}=$ 10 m. sec.$^{-1}$, $\phi=45°$ N., $\sin\alpha=10^{-1}$, $\sigma=4$) we can plot the expression (4.19) as a function of n. This is done in figure 7, where we also have multiplied by the area to obtain a unit comparable with the units in figures 6 and 7. We obtain a spectrum with a broad maximum around $n=8$. It is not too difficult to imagine that this spectrum would change into one with a structure similar to those in figures 6 and 7 when the standard values of $v_{2\,max}$ and v'_{max} were replaced by observed values. As mentioned before this would tend to give a maximum for $n=2$ or 3 and $n=6$ or 7. We may therefore say that *linear adiabatic* theory accounts for the gross features of the spectra obtained for January and April, although there are too many factors ($v_{2\,max}$, v'_{max}, and α) to make a close comparison.

Returning now to the nature of the spectra, we concluded that the maximum appearing for $n=6$ or 7 could be ascribed to the unstable baroclinic waves. In the preceding paragraphs we also pointed to the possibility that the maximum for $n=2$ or 3 could appear because of the rather great amplitude in the meridional winds for these wave numbers, if we made a comparison with an *adiabatic, linear* theory. The following question may now be asked: Would the energy conversion spectra be radically different if external effects (heating, friction) could be taken into account? Suppose for a moment that the spectrum would be changed only slightly. This would mean that the ultra-long waves ($n=1$, 2, and 3) would be self-maintaining, having an energy conversion which takes place on the same scale, which then would balance the frictional dissipation. If on the other hand, heating, for instance, will produce a system of vertical motions which are correlated with temperatures in such a way that the energy conversion is greatly reduced, we will have to postulate another mechanism responsible for the maintenance of the kinetic energy of these long waves against frictional dissipation. One possible mechanism would be a transfer of kinetic energy from higher to lower wave numbers through non-linear interaction, a mechanism which certainly must be operating in the atmosphere. It has in fact been shown by Fjørtoft [4] that a transfer of this nature takes place in a two-dimensional, non-divergent fluid. The quasi-barotropic character of the atmosphere makes it likely that the same mechanism to some extent is operating in the real atmosphere.

A definite answer to the question stated above is not easy to give, mainly because the distribution of heat sources and sinks is not known with any great accuracy. The present knowledge should, however, be sufficient to discuss at least the order of magnitude of the effect. The following section will deal with this question.

5. ON THE INFLUENCE OF DIABATIC HEATING ON THE ENERGY CONVERSION

In this section we shall first estimate the vertical velocities due to a reasonable distribution of heat sources and sinks. Next, we shall use the distribution of the vertical velocities to estimate the correction in the energy conversion.

Very little is known about the distribution of the heat sources and sinks in the vertical direction. We shall in the following consider a component of the heating function dQ/dt, which is the heating per unit mass and unit time prescribed by the formula:

$$\frac{dQ}{dt}=r\left(\frac{p}{p_0}\right)^{\delta}\sin m\,x\,. \tag{5.1}$$

The main assumption made in (5.1) is that the vertical variation of dQ/dt can be assumed to be a power function of pressure. The exponent δ determines the decrease of the heating with height. Especially on the large scale it is reasonable to assume that the main part of the heating and cooling is due to interaction between the air and the underlying surface. The heating function ought therefore to decrease with height. The parameter δ has in the following computations been set equal to 3, which gives a rather rapid decrease with height, but the value of δ is open for discussion.

In (5.1) r measures the maximum intensity of the heating at the surface ($p=p_0$). Some recent computations of the vertically averaged heating makes it possible to determine r. The distribution of heating and cooling presented by Staff Members, Academia Sinica [17] shows in January a well-defined pattern on the ultra-large scale (essentially two waves) with an amplitude approximately 10^{-5} cal. gm.$^{-1}$ sec.$^{-1}$ for the vertically averaged heating. This corresponds to a total heating of 0.4185 kj. m.$^{-2}$ sec.$^{-1}$ The total maximum heating obtained from (5.1) is

$$H=\int_0^\infty\frac{dQ}{dt}\rho dz=\frac{1}{g}\int_0^{p_0}r\left(\frac{p}{p_0}\right)^{\delta}dp=\frac{p_0}{g}\frac{r}{\delta+1}\cdot \tag{5.2}$$

Equating these two we find that

$$r\simeq 0.17\text{ kj. t.}^{-1}\text{ sec.}^{-1} \tag{5.3}$$

If, on the other hand, we take the values used by Smagorinsky [15] we find that he considers a value of 0.3 cal. cm.$^{-2}$ min.$^{-1}=\frac{1}{2}\times 0.4185$ kj. m.$^{-2}$ sec.$^{-1}$ as a representative value in winter, although an overestimate in summer. As seen, this value is only half the value computed by the Staff Members, Academia Sinica, and would correspond to $r\simeq 0.08$ kj. t.$^{-1}$ sec.$^{-1}$ Our computations are, however, linear in r, and it will be easy to find the corrections due to the assumed intensity of the heating.

The author, in collaboration with Dr. N. A. Phillips and in connection with other problems, has recently made a computation of the heating for the month of January. It is hoped that these computations can be described in detail later. It suffices here to say that the heating was computed from the thermal vorticity equation in the sta-

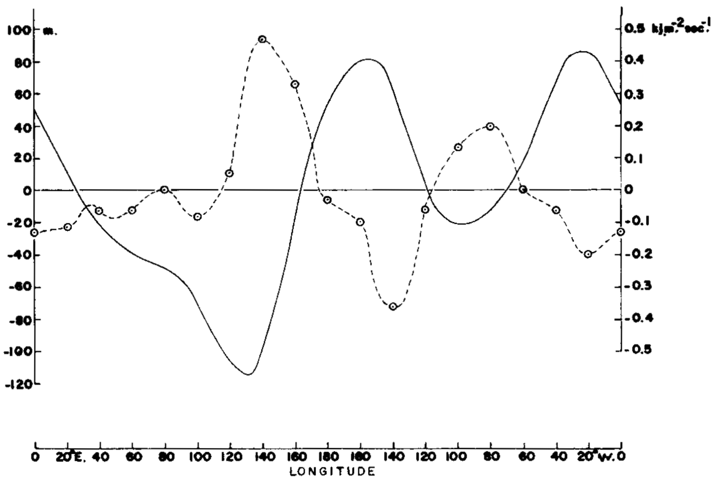

FIGURE 8.—The solid curve is the profile of the mean relative topography along 50° N. for the normal maps of January given as departures from the mean value (left scale). The dashed curve gives the latitudinal distribution of heat sources and sinks computed from the same normal maps (right scale).

tionary case in a two-parameter model paying attention to the influence of mountains and surface friction. The two-parameter model that was used was carefully constructed using the scheme described by Eliassen [3]. The normal maps prepared by Jacobs [6], Wege [20], and U.S. Weather Bureau [19] for the surfaces 1000, 850, 700, 500, 300, 200, and 100 mb. were used to define the average height field $\bar{z}$ and the thermal field h. The heating derived this way for 50° N. is reproduced in figure 8 together with the mean temperature field h.

The result of the computation is in many respects similar to the one obtained by Staff Members, Academia Sinica, although the maximum heating and cooling in our computation appears a little more toward the west than in theirs. The maxima are of the same order of magnitude.

The vertical motions produced by the heating and cooling may be found from the ω-equation. Including the diabatic heating in the thermodynamic energy equation we arrive at an ω-equation of the form:

$$\sigma\nabla^2\omega+f_0^2\frac{\partial^2\omega}{\partial p^2}=\frac{\partial}{\partial p}(\mathbf{V}\cdot\nabla\eta)-\nabla^2\left(\mathbf{V}\cdot\nabla\frac{\partial\phi}{\partial p}\right)-\frac{R}{C_p}\frac{1}{p}\nabla^2\left(\frac{dQ}{dt}\right). \tag{5.4}$$

We are here interested in only the vertical motions due to heating and cooling and shall consequently consider the following equation:

$$\sigma\nabla^2\omega+f_0^2\frac{\partial^2\omega}{\partial p^2}=-\frac{R}{C_p}\frac{1}{p}\nabla^2\left(\frac{dQ}{dt}\right). \tag{5.5}$$

In the solution of (5.5) we shall assume that $\sigma=a/p^2$. For the goodness of this approximation see a paper by the author [22]. A solution to (5.5) may now be written in the form

$$\omega(x,p)=F(p)\sin mx \tag{5.6}$$

where $F(p)$ will have to satisfy the equation

$$f_0^2\frac{d^2F}{dp^2}-m^2\frac{a}{p^2}F=\frac{R}{C_p}m^2\frac{r}{p_0^\delta}p^{\delta-1}. \tag{5.7}$$

Solutions to the homogeneous part of this equation are of the form p^α. Inserting this function in the homogeneous equation corresponding to (5.7) we find that α has to satisfy the equation

$$\alpha^2-\alpha-\frac{m^2a}{f_0^2}=0 \tag{5.8}$$

giving the two solutions

$$\alpha_{1,2}=\frac{1}{2}\left(1\pm\sqrt{1+4am^2/f_0^2}\right). \tag{5.9}$$

We find further that $F(p)=Bp^\epsilon$ is a solution to the nonhomogeneous equation provided

$$\epsilon=\delta+1,\ B=\frac{R}{C_p}\frac{m^2r}{p_0^\delta}\frac{1}{f_0^2\delta(\delta+1)-m^2a}. \tag{5.10}$$

The complete solution may therefore be written:

$$F(p)=Bp^{\delta+1}+C_1p^{\alpha_1}+C_2p^{\alpha_2}. \tag{5.11}$$

The two integration constants C_1 and C_2 are determined from the boundary conditions $\omega=0$ for $p=0$ and $p=p_0$, giving $C_2=0$ and

$$C_1=-Bp_0^{\delta+1-\alpha_1}. \tag{5.12}$$

The complete solution is therefore

$$F(p)=Bp_0^{\delta+1}\left[\left(\frac{p}{p_0}\right)^{\delta+1}-\left(\frac{p}{p_0}\right)^{\alpha_1}\right]. \tag{5.13}$$

$F(p)$ gives the vertical distribution of the vertical velocity, when it is a maximum. This distribution is given in figure 9, where we have used the following parameters: $f_0=10^{-4}$ sec.$^{-1}$, $a=10^4$m.^{2} sec.$^{-2}$, $r=0.08$kj.t.$^{-1}$ sec.$^{-1}$, $m=0.45\times10^{-6}$m.$^{-2}$ (corresponding to two waves around the hemisphere).

Our next problem is to find the energy conversion due to this vertical velocity. The most critical feature here is of course in which way the temperature (thickness) patterns are arranged relative to the heat sources and sinks and therefore also to the diabatic vertical motion. Let us write the component of the thickness field as

$$h=h_a\sin(mx+\Delta) \tag{5.14}$$

where h_a is the amplitude of the thickness field and Δ the phase-lag between the heat source and the thickness field. Note that Δ is positive if the thickness field is lagging behind the heat source field. The diabatic vertical

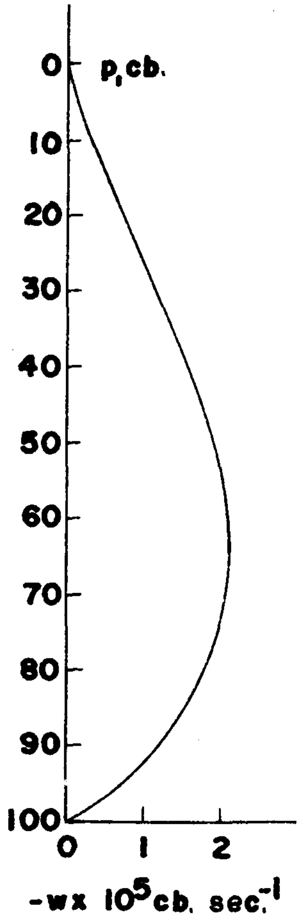

FIGURE 9.—Vertical profile of vertical velocity caused solely by heating.

velocity may be written

$$\omega = \omega_a \sin (mx + \pi) \tag{5.15}$$

where ω_a now is positive.

The energy conversion per unit area due to diabatic heating may then be computed from the expression

$$\frac{1}{L} \{P, K\} = -\frac{32}{21} \frac{1}{L} \int_0^L \omega h dx \tag{5.16}$$

which reduces to

$$\frac{1}{L} \{P, K\} = \frac{16}{21} \omega_a h_a \cos \Delta. \tag{5.17}$$

In estimating Δ it is important to note that the computations of heat sources and sinks show that the atmosphere is heated where it is cold and cooled where it is warm. This is seen from figure 8 where the heating is approximately 180° out of phase with the thickness pattern. The Academia Sinica computation shows about the same although the phase difference here is smaller. It seems therefore safe to conclude that $\cos \Delta$ for the month of January is negative for the large-scale motion. The amplitude h_a was computed as the amplitude of the second Fourier component in the thickness in figure 8.

If the heat sources and sinks were situated just below the thermal troughs and ridges, we would have $\cos \Delta = -1$ and the energy conversion could be estimated to be ($h_a \simeq 50$ m.)

$$\{P, K\}_{(2),\ \mathrm{diab.}} \simeq -9 \times 10^{-4} \text{ kj. m.}^{-2} \text{ sec.}^{-1} \tag{5.18}$$

as compared to

$$\{P, K\}_{(2),\ \mathrm{adiab.}} \simeq +1.4 \times 10^{-4} \text{ kj. m.}^{-2} \text{ sec.}^{-1} \tag{5.19}$$

Although (5.18) may be an overestimate due to the assumption that $\cos \Delta = -1$ and further to the uncertainty of the intensity and position of the heat sources and sinks, it is safe to conclude that the large energy conversion found by the adiabatic computation in winter is not real, but that the heating and cooling produce a system of vertical motion which is correlated with the mean temperature in such a way that the energy conversion from potential energy to kinetic energy is greatly reduced or perhaps even takes the opposite sign.

If this conclusion is right, it also follows that the very long waves must receive the necessary amount of kinetic energy to balance the frictional dissipation in other ways than through energy conversions from the potential energy. One possibility is, as mentioned before, that non-linear interaction between shorter and longer waves transfers energy to the long waves. This apparently means that it is necessary to consider the changes in the shorter waves in order to predict changes on the larger scale. This conclusion, if true, makes the prediction of the large-scale motion extremely difficult.

Another conclusion of importance for even short-range prediction is that any baroclinic numerical prediction model, which does not include the effect of diabatic heating, apparently converts too much potential energy into kinetic energy on the very large scale. For this reason alone it seems important to incorporate large-scale heat sources and sinks in a baroclinic model.

It should also be mentioned that diabatic heating may modify the energy spectra on the smaller scale, although the main effect there would be heating due to condensation rather than interaction with the underlying surface. The greater part of the condensation in winter is connected with the traveling waves ($n=6$ or 8). Here, however, it is most likely that the diabatic heating will work in the opposite direction because a release of condensation heat can be of significance only if a considerable amount of moisture is available, and this seems to be the case only in a warm air mass. For this scale we may therefore expect that the vertical motions produced by the heating are positively correlated with the thickness. A qualitative argument of this type indicates that the energy conversion on the scale $n=6$ or 8 may be greater than indicated in figures 5 and 6.

6. GENERAL CONCLUSIONS

On the basis of computations of energy conversions from potential to kinetic energy for individual days it was found that each individual day gave a positive conversion of potential to kinetic energy. The mean conversions for the months of January and April were 15.6×10^{-4} kj. m.$^{-2}$ sec.$^{-1}$ and 10×10^{-4} kj. m.$^{-2}$ sec.$^{-1}$, respectively. Estimates of the frictional dissipation based on these computations agree fairly well with values obtained by Pisharoty from independent data. It is found that energy conversion in the meridional planes is small in magnitude compared to the conversion in the zonal planes. The computations further show that the weak meridional circulation consists of the classical three-cell pattern. A difference between January and April is found with respect to energy conversions in the meridional planes. The meridional circulation in January has a positive energy conversion, while it is negative in April. This seems to be connected with a less well-defined direct circulation in the low and high latitudes in April.

It is further found that the energy conversion gets smaller and smaller as time progresses in the present JNWP forecasts, verifying an earlier observation that the phase-lag between the temperature and pressure fields becomes small too rapidly in these forecasts. The mean meridional circulation is maintained in the forecasts although there seems to be a shift of the pattern toward the north.

The energy conversion for different wave numbers is computed in section 4. It is found that an adiabatic, frictionless computation shows two maxima for the wavelengths corresponding to 2 and 6 or 7 waves around the hemisphere.

In section 5 it is shown that the maximum, which appears for the very long waves, most likely would disappear if heat sources and sinks could be taken into account. It seems therefore that the very long waves will receive the main part of their kinetic energy through a non-linear interaction with the shorter waves.

7. ACKNOWLEDGMENTS

The present study has benefited greatly from Professor Erik Palmén, who directed the author's attention toward the problems treated here. The author would like to thank Dr. Norman A. Phillips for discussions of the problem of the importance of diabatic heating, Mr. G. 'Arnason for permission to use his program for Fourier analysis, and Mrs. Margaret McLaughlin for the coding of the additional computations in this paper.

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CORRECTION

MONTHLY WEATHER REVIEW, vol. 87, July 1959, p. 281: In column 2, line 2, the vertical motion value should be ± 1 mm./sec.